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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Dr. J. W. L. GLAISHER, M.A., F.R.S., PRESIDENT, in the Chair.

Alexander Eugen Conrady, F.R.M.S., 104 Park Street, Camden Town, N.W.;
Henry Tresawna Gerrans, M.A., Worcester College, Oxford; and
Maurice Edmond Gheury, 23 Downshire Hill, Hampstead, N.W.,

were balloted for and duly elected Fellows of the Society.

The following Candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Henry Cunliffe Hardman, Ruskin College, Oxford (proposed by Sir R. S. Ball);
Francis Kennedy McClean, 1 Onslow Gardens, South Kensington, S.W. (proposed by Frank McClean); and
Rev. Edwin Albert Phillips, B.A., Lecturer in Mathematics, the Training College, Exeter (proposed by Arthur W. Clayden)

Seventy-two presents were announced as having been received since the last meeting, including amongst others :—

Sir N. Lockyer, Total Eclipse of the Sun, May 28, 1900: Account of Observations, presented by the Solar Physics Committee; 63 Charts of the Photographic Chart of the Heavens, presented by the French Ministry of Public Instruction.

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Jacobi's Nome (q) in Astronomical Formulæ with Numerical Tables. By R. T. A. Innes.

The Nome q, introduced by Jacobi, can be used in several astronomical formulæ, chiefly as a convenient means of calculating the values of the complete elliptic integrals. It is defined by the following equations :—

$$q = \epsilon^{-\pi\frac{K_1}{K}}$$

wherein

$$K = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{d\chi}{\sqrt{1-k^2\sin^2\chi}}\,; \qquad K_1 = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{d\chi}{\sqrt{1-k_1^2\sin^2\chi}}$$

and

$$k^2 + k_1^2 = 1.$$

We also have

$$K = 1 + \left(\frac{1}{2}\right)^2 k^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 k^4 + \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 k^6 + \text{\&c.}$$
$$= \left[1 + 2q + 2q^4 + 2q^9 + 2q^{16} + \text{\&c.}\right]^2$$

From these formulæ we may derive

$$q = \tfrac{1}{16}k^2 + \tfrac{1}{32}k^4 + \tfrac{21}{1024}k^6 + \tfrac{31}{2048}k^8 + \text{\&c.} \qquad \ldots \qquad \ldots (a)$$
$$q^{\frac{1}{4}} = \tfrac{1}{2}\sqrt{k} + 2(\tfrac{1}{2}\sqrt{k})^5 + 15(\tfrac{1}{2}\sqrt{k})^9 + 150(\tfrac{1}{2}\sqrt{k})^{13} + \text{\&c.} \quad \ldots (b)$$

Schwarz ("Formeln und Lehrsätze zum Gebrauche der elliptischen Functionen," Art. 40) remarks that the nome q and every power of that quantity of which the exponent is a positive number, may be represented by series proceeding according to powers of k, the coefficients of the different terms of these series are positive numbers, and their sum in each case is equal to unity. Unless k is very small the convergency of the two series (a) and (b) leaves something to be desired.

For practical work the series :—

$$q = \tfrac{1}{2}l + 2(\tfrac{1}{2}l)^5 + 15(\tfrac{1}{2}l)^9 + 150(\tfrac{1}{2}l)^{13} + \text{\&c.}$$

where

$$l = \frac{1-\sqrt{k_1}}{1+\sqrt{k_1}},$$

or if $k = \sin\theta$

$$l = \frac{1}{2}\,\frac{\sin^2\frac{\theta}{2}}{(1+\sqrt{\cos\theta})^2}$$

is very convergent.

As K_1 bears the same relation to k_1 that K does to k, it is

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